REVIEW OF WORK ON THE DEVELOPMENT AND APPLICATION OF METHODS OF GENERALIZED ANALYSIS

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UDC 536.24/25

The characteristics and possibilities of the methods of generalized analysis, defined as the theory of the means of universalizing quantitative investigation, are examined, together with examples of the application of these methods.

Generalized Analysis*

The development of methods of generalizing the results of numerical solutions and experiments has led to their being combined in a single system of scientific knowledge, namely generalized analysis or the study of methods of universalizing quantitative investigation, i.e., determining the most rational form in which to represent the results obtained so as to maximize their region of applicability. Generalized analysis is characterized by a single universalization principle, namely transition to new variables that include, in implicit form, the parameters of the problem, which thus lose their role of independent arguments. This principle can take two essentially different forms insofar as the parameters can be broken up into two groups of quantities very dissimilar with respect to both their physical nature and the place they occupy in the mathematical model of the process. The first group consists of the parameters that determine the material properties of importance to the process. These are a kind of physical quantity corresponding to dimensional formulas of complex structure constructed from the dimensional formulas for the original variables. The parameters in question enter into the composition of the equations of the problem. The second group contains parameters that are certain particular (so-called parametric or representative) values of the original variables known from the formulation of the problem. They serve as a means of determining specific features of the physical circumstances important for the development of the process but not depending on it and are introduced via the boundary conditions.

If the parameters entering into the composition of the new variables belong to the second group, then the transition to these variables can be made by means of a substitution of the type $z = z_0\hat{z}$, i.e., by replacing the absolute value of z by its relative value $\hat{z} = z/z_0$, where z_0 is its parametric value. This is the beginning of a path which, naturally, by logical necessity, leads directly to the set of results obtained in classical similarity theory. These results are well known and very widely employed [1]. Here, we will confine ourselves to recalling that the variables are transformed by combining all the parameters into a relatively small number of dimensionless power complexes (similarity criteria or dimensionless numbers π), which are the parameters of the problem converted to the new variables. At the same time, the conditions of uniqueness of the solution are universalized (by virtue of the obvous equation $\hat{z}_0 = z_0/z_0 = 1$). As a result, the unknown function, represented in relative form, is determined as a function of the relative independent variables and similarity criteria. On reverting to absolute values, each particular solution gives rise to a set of solutions corresponding to various parametric values of the variables, which in different combinations are separated from the criteria as individual scales. Ultimately, universalization is achieved at the level of the transition from individual phenomena to groups of similar phenomena in complete conformity with the nature of the initial operation, the significance of which consists in the elimination of such individual characteristics of the phenomenon as the absolute values of the variables and transition to relative descriptions.

* The exposition takes as its frame of reference the boundary value problem of mathematical physics.

Moscow Institute of Chemical Machine Building. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 5, pp. 717-725, November, 1987. Original article submitted March 23, 1987.

0022-0841/87/5305-1233 $12.50

The second form of transformation of the variables, based on the incorporation of parameters of the first group, leads, as in the previous case, to a transition from absolute to relative values. However, the transformation operation itself is, both technically and physically, much more complex. This is because the transformed variables and parameters are quantities that dimensionally are not at all homogeneous. The transformation is carried out as follows [2]. For each variable we introduce a special, initially undetermined value, which is called the characteristic value and denoted by z_*. Thus, the initial substitution takes the form $z = z_* z_+$, where $z_+ = z/z_*$ is a dimensionless variable. Clearly, this operation must lead to the formation of dimensionless power complexes composed of the parameters of the first group and the characteristic values of the variables and representing the parameters of the problem converted to dimensionless variables. The complexes π_* (we adopt this notation in order to stress the analogy between them and the similarity criteria π) are distinguished by the fact that their numerical values are not determinate. We require them all to reduce to unity, which means, essentially, that the quantities z_* take numerical values such that the equations are completely universalized.

Obviously, in order to satisfy this requirement it is necessary that the number r of complexes π_*, i.e., the number of equations $\pi = 1$, be not greater than the number v of variables ($r \leq v$). If $r > v$, for some of the complexes, $r - v$ in number, definite values are established, as a result of which they go over into the category of similarity criteria. The boundary conditions are also a source of criteria formation, since each parametric value of a variable, converted to dimensionless form, becomes a criterion of the type z_0/z_*. However, as distinct from the equations, the boundary conditions can be subjected to manipulation (transformation of coordinate system, discarding of weak conditions), leading to the elimination of the parametric values. Moreover, when $r < v$ the system of equations $\pi = 1$ does not determine the numerical values of all the quantities z_* and hence in the corresponding criterion it is possible to set $z_* = z_0$ and thereby reduce it to unity. Practical experience with the application of this method of universalization shows that, as a rule, its use leads to considerable or even complete universalization of the equations and sometimes, though admittedly not very often, to the solution of the problem as a whole. Everything that has so far been said about the generalized (in the relative representation) and individual solutions (in absolute quantities) remains valid, except that the relationship between them is established in terms not of parametric but of characteristic scales. From this standpoint, the two forms of universalization of quantitative investigation considered can be distinguished as the method of parametric scales and the method of characteristic scales.

The method of characteristic scales is a highly flexible one and this property, attributable to the original indeterminacy of the scales, opens up new possibilities. As noted above, the use of the method ensures a higher level of universalization and, under favorable conditions, the complete universality of the solution (although, in the language of similarity theory, the physical circumstances are not self-similar and the similarity criteria are not degenerate). Below, it will be shown that a situation in which the use of the method makes possible the determination of the form of the function in the end equation or, under more complex conditions, a significant simplification of the problem is physically real.

The following feature of characteristic scales deserves special attention. When $v \leq r$ they are formed exclusively from the parameters of the first group. This means that they are completely determined by the properties of the process itself. Thus, as distinct from the parametric values of the variables which, reflecting the external conditions, are assigned quite arbitrarily relative to the process, the characteristic values are intrinsic natural scales of the process and provide a good basis for estimating the order of the values of the dimensionless variables (under conditions such that all the complexes reduce to unity, which indicates the more or less uniform intensity of all the physical effects important to the process). If $v > k$ and some characteristic value is identified with a parametric value, then the scales may be considered natural only if it is certain that by means of this parametric value a condition of decisive importance for the development of the process is expressed.

Let us now consider some examples to illustrate the method of characteristic scales [2-5]. In the interests of convenience and the more complete formalization of the apparatus of investigation of concrete problems (one-dimensional and two-dimensional problems will be considered), we will introduce the concept of a self-similar (similar) solution, by which we understand the special situation in which it is possible to determine the form of the unknown

function in the end equation or implement a change of variables such that the two-dimensional problem is reduced to one-dimensional form.

For the existence of a self-similar solution it is sufficient that

$$k < v, \tag{1}$$

where k is the total number of dimensionless power complexes, including similarity criteria of the type z_0/z_*, less the criteria constructed wholly of parameters of the first group (a typical example is the Prandtl number) – the emergence of such criteria as arguments does not prevent the obtaining of a self-similar solution. Inequality (1) may be considered not only a sufficient but also a necessary condition of existence of a self-similar solution. In the case of the one-dimensional problem (v = 2, k = 1) the equation for the dimensionless unknown variable can be written in the form:

$$\varphi \Pi l_*^n = \varphi_+ (x/l_*;\ \pi_1,\ \pi_2,\ \ldots,\ \pi_m), \tag{2}$$

where Π is a dimensional power complex composed of parameters of the first group; l_* is the characteristic scale for extension; $\pi_1, \pi_2, \ldots, \pi_m$ are dimensionless numbers containing only parameters of the first group. The scale l_* remains indeterminate and can be chosen arbitrarily, but in this case the function ϕ_+ must be homogeneous. This property is possessed only by a power function [1]. Sometimes an equation of type (2) cannot be written for the function itself but can be written for its first derivative. In this case (when n = 1) the unknown distribution will be logarithmic. Thus, the self-similar solution of the one-dimensional problem is either a power function or a logarithmic function.

For the two-dimensional problem we can write, for example:

$$\varphi_+ = \varphi_+ (x_+,\ \tau_+;\ \pi_1,\ \pi_2,\ \ldots,\ \pi_m). \tag{3}$$

Here, from two equations for three characteristic scales (ϕ_*, τ_*, l_*) we have

$$\tau_* = \Pi_1 l_*^{n_1};\quad \varphi_* = \Pi_2 l_*^{n_2} \tag{4}$$

and

$$\varphi_+ = \varphi_+ (x/l_*,\ \tau/(\Pi_1 l_*^{n_1});\ \pi_1,\ \pi_2,\ \ldots,\ \pi_m). \tag{5}$$

The scale l_* remains indeterminate, so that Eq. (5) can be reduced to the form:

$$\varphi = \Pi_2 x^{n_2} \varphi_+ (\tau/(\Pi_1 x^{n_1});\ \pi_1,\ \pi_2,\ \ldots,\ \pi_m). \tag{6}$$

It is assumed that a solution of the problem exists and is unique; therefore relation (6) implies a transition from a partial to an ordinary differential equation.

The method of characteristic scales has definite advantages when it comes to solving optimization and technicoeconomic problems. Here, a positive effect can be achieved by going over to generalized variables and abandoning the "traditional" form. A problem typical in this respect is examined in [6], where the efficiency of convective heat-transfer surfaces of complex shape is estimated.

If we forego the use of such conventional (for channels of complex shape) concepts as the average throughput and equivalent diameter, it is possible to obtain a generalized equation for the dimensionless heat-transfer coefficient α_+ in the form:

$$\alpha_+ = \alpha/(c_p \sqrt{\rho^2 n}) = f_1 [1/(\beta \nu) \sqrt[3]{n/\rho}], \tag{7}$$

where n is the specific power expended on pumping through the gaseous heat-transfer agent. This relation enables us not only to carry out design calculations but also to estimate, in convenient form, the comparative efficiency of surfaces of complex shape. The use of Eq. (7) for determining the minimum dimensionless reduced annual consumption makes it possible to find the most efficient heat transfer surface operating regime. It has been shown that in a number of cases a rational choice of operating regime gives better results than replacing the surface in question by another known to be more efficient, while retaining the previous operating regime.

The existence of a rigorous system for obtaining generalized variables makes it possible successfully to employ the method of characteristic scales for testing the validity of theoretical hypotheses. This is of particular value in connection with the various kinds of semiempirical theories typically associated with problems of turbulence, boiling heat transfer, fluidization, etc.

In [7, 8] a phenomenological model of the phase motion in a fluidized bed is considered; the application of the method of characteristic scales to the model is demonstrated and a direct experiment – testing the necessary and sufficient conditions of similarity of two fluidized bed apparatuses – is discussed.

It is worth noting two points in particular, both of which illustrate the value of the method of characteristic scales as an investigative tool and reveal the reasons for the lack of success in modeling a fluidized bed. Firstly, it was found that the natural radial scale of the momentum transfer process in the unloaded part of the bed is commensurable with the overall dimensions of even industrial (let alone laboratory) apparatus and more often than not exceeds them. Consequently, the transfer process cannot develop in the radial direction, and the degree of "suppression" plays a decisive part. Secondly, the natural scales characterizing the grid zone depend on the hydraulics of the entire technological chain. Consequently, the hydrodynamics of the fluidized bed have a "hidden" degree of freedom, since it is part of a system and the system influences its part.

The method of characteristic scales makes possible the optimal organization of the numerical solution of problems on a computer, where a leading role is played by the development of a physicomathematical model. In fact, to the computer programmer's key question concerning the degree of accuracy with which the solution should be obtained the developer of the model replies: with an absolute accuracy of not more than 10^{-1} times the natural scale of the unknown function. The model itself does not provide greater accuracy, and a rougher solution is not acceptable. The problem of quantization of the space–time manifold should not be solved arbitrarily by the computer programmer. His natural tendency to reduce the computation time and fit the problem into the computer's memory leads to a coarsening of the cells of the space–time grid. For the model developer this means that the end result will fail to reflect the influence of those physical effects which have small natural linear and time scales. The absolute dimensions of the cells should not exceed 10^{-1} times the corresponding natural scales.

In conclusion, we note the following. If the basic equations of the problem are not given, then in constructing the complexes π (or π_*) the unknown equations can be replaced by determinant equations, i.e., by equations that determine the secondary quantities as functions of the primary quantities and are represented in the form of dimensional formulas. In this case the primary quantities not important to the problem but present in the individual complexes can be eliminated by combining the corresponding complexes.

Investigation of Momentum, Heat and Mass Transfer in Turbulent Wall Flows by Means of Dimensional Analysis

Although the universal law of turbulent friction

$$U_1/u_* = \sqrt{\lambda/8} = \varkappa^{-1}\ln(\delta u_*/\nu) + B + B_1 \tag{8}$$

derived by means of dimensional analysis has been known since the end of the thirties, the analogous universal law of turbulent heat and mass transfer was not derived until comparatively recently [9]:

$$\mathrm{Nu} = \frac{\mathrm{Re}\,\mathrm{Pr}\sqrt{\lambda/8}}{\alpha\ln(\mathrm{Re}\sqrt{\lambda/8}) + \beta(\mathrm{Pr}) + \beta_1 - \beta_2 + \beta_3\sqrt{\lambda/8}}. \tag{9}$$

Here, B_1 and β_1 are constants that depend on the type of flow (pipe, plate, etc.), and B and β(Pr) determine the velocity and temperature (or concentration) drop in the viscous sublayer and, for a rough wall, also depend on the roughness parameters. It was found that the existing experimental data, supplemented by specially designed experiments [10], make it possible to determine the values of all the constants and functions in (8) and (9). For example, the turbulent Prandtl number Pr_t in a logarithmic sublayer was found to be equal to 0.85 [9], so that $\alpha = \mathrm{Pr}_t/\varkappa = 2.1$, and $\beta(\mathrm{Pr}) = (3.85\mathrm{Pr}^{1/3} - 1.3)^2 + 2.1\ln\mathrm{Pr}$. In this case Eq. (9) made possible a perfectly satisfactory description of, for example, experiments on the heat and mass transfer associated with developed turbulent flow in smooth pipes carried out on the intervals $5\cdot10^3 \lesssim \mathrm{Re} \lesssim 2\cdot10^6$ and $10^{-2} \lesssim \mathrm{Pr} \lesssim 10^5$. Moreover, a detailed study of the characteristics of flow over rough surfaces has made it possible [11, 12] to determine the roughness functions B and β in Eqs. (8) and (9) and on that basis to develop, for technically important cases of two-dimensional roughness, a complete method of calculating the coefficients of friction λ and heat and mass transfer Nu.

On the basis of the results obtained, certain additional ideas concerning the development of the thermal boundary layer in a stabilized turbulent flow [13, 14], and specially designed experiments [15], it has also proved possible to develop a method of calculating Nu on the thermal entrance length [16] valid over a broad range of variation of the geometric and regime parameters for various boundary conditions.

The subsequent development of the approach described has involved the study of more complex cases of turbulent flow subjected to various influences. Thus, for example, it has been found that accelerating or decelerating flows give rise to a "gradient sublayer," the self-similar structure of the turbulence in which is determined no longer by the parameter u_* but by the kinematic coefficient of the longitudinal pressure gradient $\rho^{-1}dp/dx$. Here, too, on the basis of dimensional considerations it has proved possible not only to obtain the general form of the velocity profiles and the law of friction of gradient flows against an underlying surface [17, 18] but also to study the fine structure of the turbulence in the gradient sublayer [19, 20]. Comparison with the available experimental data again proved perfectly satisfactory.

We note, moreover, that dimensional considerations, supplemented by an hypothesis perfectly analogous to the assumption on which the theory of small-scale turbulence is based, have made it possible [20-22] to describe the shape of the turbulent fluctuation spectra in a region of anisotropic eddies (much smaller, however, than the external scale of the flow) and have thereby opened up the path for the study of the large-scale eddy structure that determines the momentum, heat and mass transfer in real flows.

Generalized Relations for the Transport Properties of Amorphous Materials

It is not uncommon to encounter problems in which the concept of similarity is restricted to finding relations linking the transport properties of a body with certain dimensionless parameters determined by the geometric structure of the material and the dynamic characteristics of the interaction between the elements of the structure and, moreover, with the external conditions (so-called thermodynamic similarity). Under the conditions of application of the methods of generalized analysis to the study of transport properties this situation arises when the basic equations of the process and the conditions at the boundary are missing from the formulation of the problem itself, but at the same time, all the parameters of importance to the process, determined from a detailed study of the physical structure of the object of investigation, are known.

The methods of thermodynamic similarity can be used for investigating the transport properties of a material in generalized form only on the assumption that the thermodynamic parameters of a certain special fundamental (critical) state, used as the parametric scales, are known. However, in the case of amorphous materials, in constructing the generalized relations it is not possible to base oneself on scales of this kind owing to the inaccessibility of the critical state of these materials. At the same time, amorphous materials have another special state – the glass transition point or solid/liquid interface. Since the parameters of this point must be determined experimentally, the parametric scales are not directly known. Therefore the general relations are constructed in two stages: (a) the relations between the parameters of the glass transition point, represented in dimensionless form by means of characteristic scales, are found; (b) general relations are constructed for the properties investigated using as parametric scales the values of the glass transition point parameters obtained by calculation. This method of construction leads to completely universal generalized equations for the transport properties of amorphous substances.

Experimental data on the thermal conductivity, diffusion and heat capacity of approximately 80 polymers and on the thermal conductivity and diffusion of about 100 silicate glasses, obtained over a broad temperature interval, have been analyzed in accordance with this scheme. The discrepancy between the calculated and the experimental values of the properties studied lies within the limits of experimental error [23, 24].

Intensification of Convective Heat Transfer by Creating Pressure Nonuniformities

The scientific content of the heat transfer intensification problem, in accordance with its physical nature, reduces to the creation in the flow of temperature and velocity distri-

butions such that the motion of the elements of the moving medium involves more transfer of heat than of momentum [25]. The corresponding physical situation is very unusual, highly complex, and can only be created artificially if the development of the process is carefully controlled. These considerations underlie a special trend in the solution of the convective heat transfer intensification problem based on the idea of acting on the micro- and macrostructure of the flow by artificially creating pressure nonuniformities by means of specially profiled surfaces [26].

In practice, this principle is embodied in flow with a longitudinal alternating pressure gradient in the form of an alternation of divergent and convergent channel flows; in flow with a succession of standing eddies; in the parallel motion of two flows – one with a longitudinal alternating pressure gradient and another in which the gradient is practically zero; in the parallel motion of two flows with longitudinal alternating pressure gradients whose pressure fields are mutually displaced by half a wavelength; and in flow with a series of separation zones forming a sequence of regions of divergent and convergent channel flow.

The principle has actually been applied in the following structural forms: surfaces with alternating plane diffusers and contractions, surfaces with crescent-shaped projections designed to create a series of standing eddies that reproduce the hydrodynamic conditions characteristic of geometric diffusers; surfaces with lengthwise wavy finning; surfaces with perforated lengthwise wavy finning; and surfaces with angular perforated adapters.

All these cases involve the creation of specific pressure nonuniformities which generate secondary flows in the form of local transverse motions that lead to intense mass transfer between the flow core and the wall zone [27] or in the form of discrete alternate injection and suction in the boundary layer zone [28].

For the time being, we are still a long way from reaching that level of understanding that would enable us to construct a quantitative theory of the transfer processes in such complex systems as the surfaces that create the above-mentioned pressure nonuniformities. Therefore direct experiment is being used as the basic quantitative method of investigation, and the theory of similarity as a means of generalizing and analyzing the experimental data. In order to construct a rational quantitative theory it will be necessary to accumulate a very large volume of experimental facts and physical ideas.

The extensive experimental material on the heat transfer and drag of the surfaces developed, in both gas and liquid flows, has provided a reliable foundation for engineering calculations and design [29-36]. In efficiency these surfaces surpass known surfaces of the same kind and can be confidently recommended for industrial use [28-31].

NOTATION

c_p, specific heat at constant pressure; k, r, and v, numbers of criteria, complexes and variables z; ℓ, an extension; n, an exponent or specific power; p, pressure; x and τ, arguments; z, a variable (argument or function); α, heat-transfer coefficient; β, surface compactness coefficient; δ, layer thickness; κ, Prandtl–Karman constant; λ, coefficient of friction; ρ, density; ν, kinematic viscosity coefficient; Re, Reynolds number; Nu, Nusselt number. Indices: 0, parametric value; *, characteristic value; +, dimensionless value; r, relative value; t, turbulent value.

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